Chapter 5

Polynomials Part A

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This book is a work in progress.

5.1 The Distributive Property

1 Categorize each polynomial by its degree (constant, linear, quadratic, etc.) and by its number of terms (monomial, binomial, trinomial, etc.) If the expression is not a polynomial, explain why.

a	$2x^2 + 4x - 7$	**b**	$x^3 - 17x$	**c**	$x^3 - 5x^2 + 9$
d	$9x + 3$	**e**	$2^x + 5x^3$	**f**	15
g	$-x^4 + x^2 - 1$	**h**	$22x^5$	**i**	$3x^2 + 4\sqrt{x} - 9$
j	$2 + 3x + 5x^4 - 7x^3$	**k**	$x^3 + 5x^2 - 8x - 2$	**l**	$-\pi x^3$
m	$\sqrt{7}x^3 - x + 15$	**n**	$-x^2 + \dfrac{3}{x}$	**o**	$9x^4 - x^3 + 2x$

2 Distribute the following expressions.

a	$3(7x + 2)$	**b**	$9(3x - 1)$	**c**	$5(4x - 7)$
d	$a(2a - 5)$	**e**	$2b(b - 13)$	**f**	$-3c(7c - 10)$
g	$3x^2(2x + 11)$	**h**	$-2x^3(8x^2 - 7x)$	**i**	$-5x(x^4 + 12x^3)$
j	$(m - 3)(m + 5)$	**k**	$(n + 7)(n + 2)$	**l**	$(p - 6)(p - 11)$
m	$(x + 9)(x + 12)$	**n**	$(x - 3)(x - 4)$	**o**	$(x + 5)(x - 13)$
p	$(x - 7)(2x + 5)$	**q**	$(3y - 4)(y - 9)$	**r**	$(5z + 2)(7z + 3)$
s	$(x + 2)(x^2 + 3x - 1)$	**t**	$(x^2 + 5x)(x^3 + 2x + 4)$	**u**	$(3x - 4)(7x^5 - 5x^3 + 2x^2)$

3 Factor the following by identifying the greatest common factor.

a	$10x + 12$	**b**	$18x - 30$
c	$3t^2 - 14t$	**d**	$24u^4 + 9u^3$
e	$25v^5 - 15v$	**f**	$14x^3 - 21x^2 + 28x - 35$
g	$72x^4 + 27x^3 - 18x^2 + 54x$	**h**	$24x^7 - 40x^5 + 48x^4$
i	$60a^3b^4c^7 + 20a^5b^2c^5 + 70a^9bc^6$	**j**	$18x^3y^4z - 24y^6z^2 + 33x^2y^5z^4$
k	$12p^2q^2r^6 - 24pq^2r^4 + pq^3r^4 + 36pqr^6$	**l**	$42x^2y^3z^4 - 84x^4y^2z^3 - 210x^3y^4z^2 + 294xyz$

Further Practice

- Distribute any monomial multiplied by any polynomial. Then have a partner factor it. Ideally, since distributing and factoring are opposite processes, they *should* return to your original expression.
- You might find that the previous suggested method doesn't always work. Explain under what circumstances factoring would *not* result in the same expression that was created before distributing.

5.2 The Zero Product Property

1 Use the zero product property to solve the following linear equations.

a $3x = 0$ **b** $-5x = 0$ **c** $12(x - 7) = 0$

d $4(x + 5) = 0$ **e** $-2(4x + 11) = 0$ **f** $5(2x - 17) = 0$

2 Use the zero product property to solve the following factored quadratic equations.

a $(x - 7)(x - 11) = 0$ **b** $(x - 1)(x + 8) = 0$ **c** $3x(x - 5) = 0$

d $-7x(x + 13) = 0$ **e** $5(x + 3)(x + 13) = 0$ **f** $-2(x + 5)(x - 17) = 0$

g $(2x - 1)(x - 5) = 0$ **h** $8x(3x + 7) = 0$ **i** $(3x - 4)(x + 11) = 0$

j $2(5x + 2)(x - 4) = 0$ **k** $-9x(12 - 5x) = 0$ **l** $-3(4x + 3)(x + 7) = 0$

m $(5x - 2)(7x + 4) = 0$ **n** $(3x + 9)(5x - 9) = 0$ **o** $3(5 - 8x)(2x + 7) = 0$

3 Use the zero product property to solve the following factored polynomial equations.

a $(x - 3)(x - 4)(x - 9) = 0$ **b** $(x + 7)(x - 2)(x + 5) = 0$

c $(2x - 5)(x + 6)(x - 1) = 0$ **d** $(3x + 7)(2x - 1)(x - 5) = 0$

e $x(x - 7)(2x + 11)(x - 3) = 0$ **f** $(7x - 5)(10x + 23)(x - 1)(x + 1) = 0$

4 Find an equation for each set of solutions given below. You should only use integers, not fractions or decimals, in your equations.

a $x = 3$ or $x = 10$ **b** $x = -7$ or $x = -4$ **c** $x = -6$ or $x = 1$

d $x = 0$ or $x = 2$ **e** $x = -9$ or $x = 0$ **f** $x = \frac{4}{7}$ or $x = 5$

g $x = -6$ or $x = \frac{11}{3}$ **h** $x = -0.5$ or $x = 7$ **i** $x = 1.5$ or $x = -0.75$

5 Solve the following equations by factoring an expression equal to zero.

a $x^2 - 3x = 0$ **b** $x^2 + 7x = 0$ **c** $x^2 = 5x$

d $x^2 = -3x$ **e** $20x^2 - 4x = 0$ **f** $7x^2 + 35x = 0$

g $3x^2 = 15x$ **h** $2x^3 - 12x^2 = 0$ **i** $7x^5 + 21x^4 = 0$

6 For each of the following functions

i find the zeros of the function,

ii find the minimum or maximum of the function, and

iii use this information to sketch a graph of the function.

a $f(x) = (x - 4)(x - 12)$ **b** $f(x) = -\frac{1}{3}(x + 8)(x - 4)$

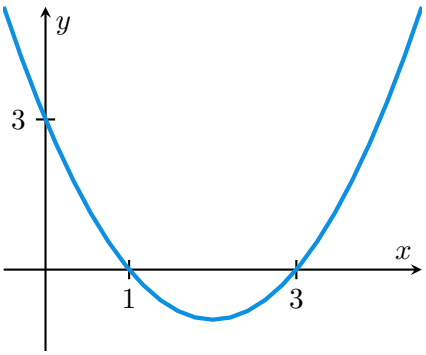
c $f(x) = -\frac{1}{2}(x + 3)(x - 5)$ **d** $f(x) = 2(x - 4)(x - 9)$

e $f(x) = (2x - 7)(2x + 3)$ **f** $f(x) = (3x - 4)(6x + 7)$

g $f(x) = 8x^2 - 4x$ **h** $f(x) = -3x^2 + 5x$

7 For each graph or description, determine the function in factored form.

a



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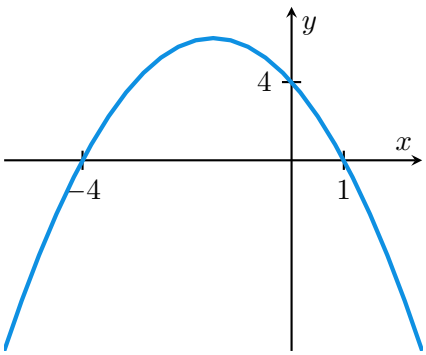

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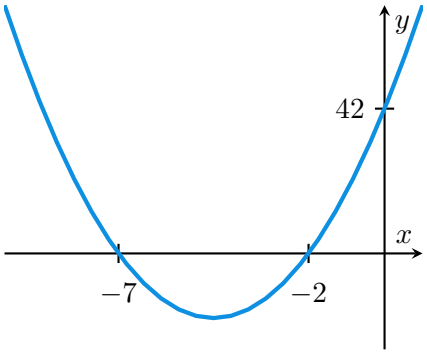
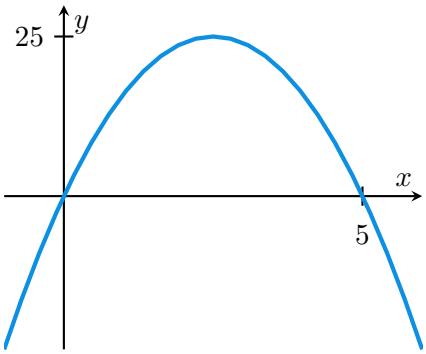


d



e Graph has x-intercepts at $(3, 0)$ and $(8, 0)$, and a y-intercept at $(0, -24)$.

f Graph has x-intercepts at $(-2, 0)$ and $(6, 0)$, and passes through $(1, 30)$.

g Graph passes through the origin and $(-8, 0)$, and it's minimum value is $y = -12$.

h Graph has x-intercepts at $(-7, 0)$ and $(-1, 0)$, and both coordinates of the vertex are the same.

5.3 Differences of Squares and Perfect Squares

1 Distribute each of the following expressions.

a $(x+3)(x-3)$ **b** $(x+5)(x-5)$ **c** $(x-11)(x+11)$

d $(x-7)(x+7)$ **e** $(x+13)(x-13)$ **f** $(x-20)(x+20)$

g $(9x-2)(9x+2)$ **h** $(3x+1)(3x-1)$ **i** $(5x-4)(5x+4)$

j $(2x+5)(2x-5)$ **k** $(3x-4)(3x+4)$ **l** $(8x+13)(8x-13)$

2 Distribute each of the following expressions.

a $(x+6)^2$ **b** $(x+8)^2$ **c** $(x-9)^2$

d $(x+21)^2$ **e** $(x-14)^2$ **f** $(x-17)^2$

g $(6x-1)^2$ **h** $(5x+1)^2$ **i** $(3x-2)^2$

j $(3x+5)^2$ **k** $(4x-7)^2$ **l** $(7x-10)^2$

3 Factor the following expressions.

a x^2-81 **b** x^2-36 **c** x^2-144

d x^2-225 **e** x^2-1 **f** x^2-81

g $25x^2-49$ **h** $9x^2-25$ **i** $16x^2-121$

j $16x^2-169$ **k** $25x^2-36$ **l** $100x^2-225$

4 Factor the following expressions.

a $x^2+10x+25$ **b** $x^2-22x+121$ **c** $x^2-8x+16$

d x^2+6x+9 **e** x^2+2x+1 **f** $x^2-30x+225$

g $4x^2+20x+25$ **h** $9x^2+42x+49$ **i** $100x^2-40x+4$

j $49x^2-42x+9$ **k** $81x^2-36x+4$ **l** $16x^2+88x+121$

5 Solve the following equations by factoring. You may need to factor using a GCF first.

a $x^2-64=0$ **b** $x^2-100=0$ **c** $x^2+14x+49=0$

d $x^2-6x=-9$ **e** $x^2+1=2x$ **f** $x^2=256$

g $5x^2-180=0$ **h** $2x^2+20x+50=0$ **i** $3x^2+147=42x$

j $4x^2-20x+25=0$ **k** $64x^2-121=0$ **l** $100x^2+180x+81=0$

m $24x^2-150=0$ **n** $147x^2-126x+27=0$ **o** $72x^2+108x+81=0$

6 Each of the expressions below is a perfect square. Find the value of n, then write the expression in factored form.

a $x^2+12x+n$ **b** $x^2-14x+n$ **c** $x^2-32x+n$

d $16x^2+72x+n$ **e** $25x^2+90x+n$ **f** $64x^2-48x+n$

g $x^2+nx+144$ **h** $x^2-nx+256$ **i** $9x^2+nx+196$

j $nx^2-60x+25$ **k** $nx^2+198x+81$ **l** $nx^2-338x+n$

7 For each of the following functions, use the difference of squares rule to write it in factored form.

a $f(x) = (x-8)^2 - 25$ **b** $f(x) = (x+4)^2 - 36$ **c** $f(x) = (x-9)^2 - 81$

d $f(x) = -(x-7)^2 + 81$ **e** $f(x) = -(x+9)^2 + 9$ **f** $f(x) = -(x+2)^2 + 64$

g $f(x) = 3(x-2)^2 - 12$ **h** $f(x) = -5(x+10)^2 + 45$ **i** $f(x) = 7(x-3)^2 - 1008$

8 Sketch a graph of the following functions, factoring if necessary.

a $f(x) = (x-1)^2$ **b** $f(x) = (x-3)^2$ **c** $f(x) = (x+7)^2$

d $f(x) = x^2 + 16x + 68$ **e** $f(x) = x^2 + 6x + 9$ **f** $f(x) = x^2 - 24x + 144$

g $f(x) = 5x^2 + 30x + 45$ **h** $f(x) = 2x^2 - 36x + 162$ **i** $f(x) = -3x^2 - 12x - 12$

j $f(x) = -16x^2 - 48x - 36$ **k** $f(x) = 50x^2 - 40x + 8$ **l** $f(x) = -12x^2 - 84x - 147$

5.4 Factoring Quadratic Expressions

1 Factor the following expressions by first factoring pairs.

a $x^2+3x+4x+12$ **b** $x^2+7x+2x+14$ **c** $x^2+2x-11x-22$

d $x^2-8x-6+48$ **e** $x^2-x+6x-6$ **f** $x^2-9x+5x-45$

g $2x^2+14x+5x+35$ **h** $3x^2-12x+8x-32$ **i** $8x^2-40x+2x-10$

j $7x^2-56x-12x+96$ **k** $5x^2+45x-2x-18$ **l** $4x^2+44x-12x-132$

2 Factor the following trinomials.

a $x^2+8x+15$ **b** $x^2+8x+12$ **c** $x^2+12x+27$

d $x^2-9x+20$ **e** $x^2-11x+18$ **f** $x^2-10x+24$

g $x^2+3x-10$ **h** x^2+x-42 **i** $x^2+3x-108$

j $x^2-4x-32$ **k** $x^2-4x-21$ **l** $x^2-6x-72$

3 For each function below,

i determine its zeros, and

ii sketch its graph, showing all intercepts and the vertex.

a $f(x)=x^2+7x+10$ **b** $f(x)=x^2-2x-8$

c $f(x)=x^2-9x+14$ **d** $f(x)=x^2-11x-26$

e $f(x)=2x^2+22x+60$ **f** $f(x)=-2x^2+8x+10$

g $f(x)=3x^2+9x-120$ **h** $f(x)=11x^2-99x-1496$

i $f(x)=\frac{1}{2}x^2+\frac{9}{2}x+9$ **j** $f(x)=\frac{3}{4}x^2+\frac{57}{4}x+63$

k $f(x)=\frac{2}{3}x^2+\frac{2}{3}x-8$ **l** $f(x)=\frac{7}{5}x^2-\frac{61}{5}x+8$

4 Factor each of the following expressions.

a $2x^2+13x+20$ **b** $5x^2+16x+3$ **c** $3x^2-16x+5$

d $2x^2-17x+21$ **e** $7x^2+15x-18$ **f** $2x^2+2x-40$

g $6x^2+8x+2$ **h** $10x^2+69x-7$ **i** $4x^2+8x+3$

j $6x^2+11x-10$ **k** $12x^2+23x-24$ **l** $45x^2+34x-24$

5 For each function below,

i determine its zeros, and

ii sketch its graph, showing all intercepts and the vertex.

a $f(x)=2x^2+10x+12$ **b** $f(x)=5x^2-11x+2$

c $f(x)=3x^2+16x-12$ **d** $f(x)=6x^2-19x-11$

e $f(x)=8x^2+30x+7$ **f** $f(x)=15x^2+4x-11$

g $f(x)=10x^2-23x+12$ **h** $f(x)=20x^2-9x-18$

i $f(x)=28x^2+113x-30$ **j** $f(x)=6x^2+5x-56$

6 Solve the following equations.

a $7x^2 - 63 = 0$

b $5x^2 + 20x + 20 = 0$

c $-2x^2 + 16x - 30 = 0$

d $-6x^2 - 54x = 0$

e $4x^2 - 12x - 112 = 0$

f $9x^2 - 42x + 49 = 0$

g $4x^2 - 25 = 0$

h $12x^2 - 54x = 0$

i $-4x^2 + 17x + 15 = 0$

j $-20x^2 + 112x + 48 = 0$

k $16x^2 - 56x - 392 = 0$

l $48x^2 - 120x + 75 = 0$

m $-20x^2 + 12x = 0$

n $10x^2 - 26x = 0$

o $-147x^2 + 12 = 0$

p $-60x^2 + 18x + 6 = 0$

q $-50x^2 - 40x - 8 = 0$

r $30x^2 + 76x - 64 = 0$

Further Practice

Remember that factoring is the opposite process to distributing. It's relatively simple to create your own problems by multiplying a pair of linear binomials. Trade them with a partner, and compare your answers with your partner's original expressions. Try a variety of difficulties, including monic and non-monic expressions.

5.5 Completing the Square

1 Solve the following equations.

a $(x-5)^2-20=0$ **b** $(x+8)^2-50=0$ **c** $(x-12)^2-72=0$

d $(x+7)^2-45=0$ **e** $(x-13)^2-12=0$ **f** $(x+3)^2-300=0$

2 Find the value of c which makes the following expressions a perfect square, and factor the expression.

a $x^2+14x+c$ **b** $x^2+32x+c$ **c** $x^2-10x+c$

d $x^2-22x+c$ **e** $x^2-40x+c$ **f** $x^2+72x+c$

g x^2+9x+c **h** $x^2+17x+c$ **i** x^2-3x+c

j $x^2-11x+c$ **k** $x^2+21x+c$ **l** $x^2-35x+c$

3 Solve the following equations by completing the square.

a $x^2+12x+27=0$ **b** $x^2-6x-7=0$ **c** $x^2+10x-56=0$

d $2x^2-16x+24=0$ **e** $5x^2-80x+315=0$ **f** $4x^2+56x-128=0$

g $x^2-15x+50=0$ **h** $x^2+15x+36=0$ **i** $x^2+3x-28=0$

j $6x^2-60x-144=0$ **k** $3x^2-39x+36=0$ **l** $2x^2-26x-28=0$

4 Solve the following equations.

a $x^2-6x+4=0$ **b** $x^2+8x+14=0$ **c** $x^2+18x+78=0$

d $x^2-3x+1=0$ **e** $x^2+7x+7=0$ **f** $x^2-9x+12=0$

5 Write the following functions in vertex form. Then sketch a graph of the function.

a $f(x)=x^2-2x-7$ **b** $f(x)=x^2+10x+13$ **c** $f(x)=x^2-16x+44$

d $f(x)=x^2+9x+9$ **e** $f(x)=x^2-13x+11$ **f** $f(x)=x^2+11x-17$

6 For each pair of solutions, create a matching equation.

a $-5\pm\sqrt{2}$ **b** $3\pm\sqrt{3}$ **c** $-7\pm\sqrt{7}$

d $4\pm2\sqrt{3}$ **e** $-9\pm5\sqrt{2}$ **f** $5\pm4\sqrt{5}$

7 Find the inverse of each function. Restrict the domain of the function as appropriate, and state the domain and range of both the function and its inverse.

a $f(x)=x^2-6x-27$ **b** $f(x)=x^2+10x+21$ **c** $f(x)=x^2+2x-24$

d $f(x)=x^2-5x+6$ **e** $f(x)=x^2+7x-44$ **f** $f(x)=x^2-3x-10$

g $f(x)=x^2+8x+25$ **h** $f(x)=x^2-6x+17$ **i** $f(x)=x^2+5x+12$

5.6 The Quadratic Formula

1 Solve the equation $ax^2 + bx + c = 0$ by completing the square. If you're feeling brave, ignore the steps below and attempt to solve it yourself. Otherwise, follow these steps.

a Write down an equivalent equation in the form $x^2 + mx + n = 0$. Note that m and n will be expressions, not numerals!

b Complete the square and find the missing part of $\left(x + \frac{b}{2a}\right)^2 + \cdots = 0$

c Show that $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$.

d Show that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

2 Determine the number of solutions for each equation. Then solve the equations, if possible. State the solutions in exact simplified form, and in approximate decimal form.

a $2x^2 + 2x - 7 = 0$ **b** $-3x^2 - 4x + 5 = 0$ **c** $x - 2x^2 - 2 = 0$

d $2x^2 + 2 = 4x$ **e** $2x^2 + x + 6 = 5x + 9$ **f** $2x^2 + 10x + 7 = 17$

3 Sketch graphs for each of the following functions, showing all important points.

a $f(x) = 3x^2 + 4x - 5$ **b** $f(x) = -x^2 + 7x - 6$ **c** $f(x) = \frac{1}{2}x^2 - 3x + 3$

d $f(x) = -4x^2 - 8x - 7$ **e** $f(x) = \frac{1}{5}x^2 - x + 4$ **f** $f(x) = \frac{1}{4}x^2 + \frac{5}{2}x + \frac{25}{4}$

4 Solve the following equations, using an algebraic method or the quadratic formula as appropriate. Use technology to check your answers.

a $x^2 + 3x - 10 = 0$

b $2x^2 + 5x - 7 = 0$

c $x^2 - 4x - 7 = 0$

d $-2x^2 - 7x + 8 = 0$

e $5x^2 - 20x + 15 = 0$

f $0.5x^2 - 8x - 21 = 0$

g $-4x^2 - 25x + 63 = -x + 7$

h $\frac{1}{2}x^2 - 3x - 9 = 4x + 5$

i $x^2 - 14x + 49 = 5$

j $2x^2 + 20x + 50 = 4$

k $-x + 15 = x^2 - 3x + 7$

l $3x - 20 = -x^2 + 3x - 9$

m $\frac{1}{3}x^2 + \frac{2}{3}x - \frac{7}{3} = 0$

n $\frac{1}{5}x^2 + \frac{8}{5}x + 3 = 0$

o $-\frac{1}{4}x^2 + \frac{7}{4}x + \frac{9}{2} = 0$

p $\frac{5}{4}x^2 - 7x - \frac{3}{4} = 0$

q $-\frac{1}{6}x^2 + 2x - \frac{5}{6} = -\frac{1}{2}x + 3$

r $-\frac{7}{3}x^2 + \frac{24}{3}x + 9 = -\frac{1}{6}x + 7$

Further Practice

Choose your own values for a, b and c in the function $f(x) = ax^2 + bx + c$. Determine the number of solutions that $f(x) = 0$ has, and solve the equation if possible. Use technology to check your answers.